

## Integration

### 3.1 Integration as the Inverse of Differentiation, Integration of $ax^n$ and integration of the Functions of the Sum/Difference of Algebraic Terms

$$\int adx = ax + C$$

Example

$$\int 2dx = 2x + C$$

#### Exercise 3.1:

1. Find

a.  $\int 4dx$

b.  $\int 120dx$

c.  $\int -12dx$

d.  $\int \frac{2}{3}dx$

e.  $\int -\frac{1}{4}dx$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Example

1.  $\int x^2 dx = \frac{x^3}{3} + C$

2. Find

a.  $\int x^4 dx =$

b.  $\int x dx =$

c.  $\int x^{-2} dx =$

d.  $\int x^{\frac{1}{2}} dx =$

e.  $\int x^{\frac{4}{5}} dx =$

Example

$$\int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-4}}{-4} + C$$

f.  $\int \frac{1}{x^3} dx =$

g.  $\int \frac{1}{x^7} dx =$

Example

$$\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2x^{\frac{3}{2}}}{3} + C$$

h.  $\int \sqrt{x} dx =$

i.  $\int \sqrt{x^3} dx =$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

Example

$$\int 2x^3 dx = \frac{2x^4}{4} + C = \frac{x^4}{2} + C$$

3. Find

a.  $\int 3x^5 dx =$

b.  $\int -7x^3 dx =$

c.  $\int -130x^{550} dx =$

d.  $\int \frac{3}{4}x^2 dx =$

e.  $\int -\frac{7}{15}x^6 dx =$

f.  $\int \frac{5x^4}{9} dx =$

g.  $\int -\frac{2x^5}{7} dx =$

h.  $\int \frac{x^{-3}}{2} dx =$

l.  $\int -\frac{5}{7} x^{\frac{1}{3}} dx =$

i.  $\int \frac{3x^{-2}}{5} dx =$

m.  $\int \frac{4}{7} x^{-\frac{3}{4}} dx =$

j.  $\int \frac{4x^{\frac{1}{2}}}{11} dx =$

Example

$$\begin{aligned} \int \frac{2}{3x^5} dx &= \int \frac{2}{3} x^{-5} dx = \frac{2}{3} \left( \frac{x^{-4}}{-4} \right) + C \\ &= \frac{\cancel{2}}{3} \left( \frac{x^{-4}}{\cancel{-4}} \right) + C = \frac{x^{-4}}{-6} + C \end{aligned}$$

n.  $\int \frac{2}{x^2} dx =$

k.  $\int \frac{2x^{\frac{2}{3}}}{3} dx =$

o.  $\int \frac{7}{x^3} dx =$

p.  $\int \frac{-2}{x^3} dx =$

Example

$$\begin{aligned}\int \sqrt{2x} dx &= \int \sqrt{2} x^{\frac{1}{2}} dx \\ &= \sqrt{2} \left( \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C = \frac{2\sqrt{2}x^{\frac{3}{2}}}{3} + C\end{aligned}$$

t.  $\int 2\sqrt{x} dx =$

q.  $\int \frac{2}{5x^5} dx =$

u.  $\int -7\sqrt{x^3} dx =$

r.  $\int -\frac{3}{4x^7} dx =$

v.  $\int \frac{2}{3}\sqrt{x} dx =$

s.  $\int \frac{5}{7x^6} dx =$

w.  $\int \sqrt{3x} dx =$

x.  $\int \sqrt{4x^5} dx =$

$$\int (u \pm v) dx = \int u dx \pm \int v dx$$

$u$  and  $v$  are functions in  $x$

**Example**

$$\begin{aligned} 1. \int 3x^2 + 2x dx &= \int 3x^2 dx + \int 2x dx \\ &= \frac{3x^3}{3} + \frac{2x^2}{2} + C = \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + C \\ &= x^3 + x^2 + C \end{aligned}$$

y.  $\int \sqrt[3]{6x^2} dx =$

4. Find

a.  $\int x^3 + 2 dx$

b.  $\int x^2 + 2x dx$

c.  $\int 2x^2 - 5x^3 + 1 dx$

z.  $\int \sqrt[4]{2x} dx =$

d.  $\int 12x^7 - 8x^{-3} dx$

e.  $\int \frac{1}{2}x^3 + 9x^8 dx$

f.  $\int \frac{1}{x^3} + \frac{2}{x^4} dx$

Example

$$\begin{aligned}\int (x+2)(3x+1)dx &= \int 3x^2 + 7x + 2dx \\ &= \int 3x^2 dx + \int 7x dx + \int 2dx \\ &= \frac{3x^3}{3} + \frac{7x^2}{2} + 2x + C \\ &= x^3 + \frac{7x^2}{2} + 2x + C\end{aligned}$$

i.  $\int (2x+3)^2 dx$

g.  $\int \frac{1}{2x^3} + \frac{9}{2}x^8 dx$

j.  $\int x^2(5x+3)dx$

h.  $\int \frac{2}{3x^5} + 9x^{\frac{1}{2}} + 3dx$

k.  $\int (3x+1)(x^2-2)dx$

l.  $\int x(x^3 + \frac{3}{x})dx$

Example

$$\int \frac{3x^3 + x^2 - x}{x} dx = \int 3x^2 + x - 1 dx$$

$$= \int 3x^2 dx + \int x dx - \int 1 dx$$

$$= \frac{3x^3}{3} + \frac{x^2}{2} - x + C$$

$$= x^3 + \frac{x^2}{2} - x + C$$

m.  $\int (x^2 + 5)^2 dx$

o.  $\int \frac{x^2 - 3x}{x} dx$

p.  $\int \frac{x^5 - 3x^3 + 5}{x^2} dx$

n.  $\int (x+1)(x^2 + 3x - 2) dx$

q.  $\int \frac{(2x-1)^2}{x} dx$

r.  $\int \frac{(2x^3 - 2)(x + 5)}{x^2} dx$

## 3.2 Integration by Substitution

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C$$

Example

$$\begin{aligned} \int (3x + 5)^3 dx &= \frac{(3x + 5)^4}{3(4)} + C \\ &= \frac{(3x + 5)^4}{12} + C \end{aligned}$$

### Exercise 3.2:

1. Find

a.  $\int (x - 1)^5 dx$

s.  $\int \frac{(x^2 - 3x)(x + 5)}{x^3} dx$

b.  $\int (5x - 9)^3 dx$

c.  $\int (12x + 3)^7 dx$



d.  $\int (2x-18)^{-2} dx$

i.  $\int \sqrt{2x+5} dx$

e.  $\int (7x+6)^{\frac{5}{2}} dx$

j.  $\int \sqrt{9x-4} dx$

f.  $\int \frac{1}{(2x-3)^2} dx$

g.  $\int \frac{2}{(5x+7)^3} dx$

h.  $\int \frac{2}{3(9x-2)^5} dx$