

Add Maths Formulae List: Form 4 (Update 18/9/08)

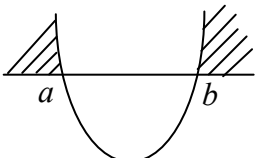
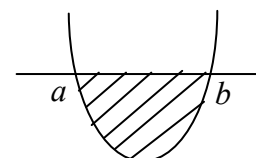
01 Functions

Absolute Value Function	Inverse Function
$f(x) \begin{cases} f(x), & \text{if } f(x) \geq 0 \\ -f(x), & \text{if } f(x) < 0 \end{cases}$	<p>If $y = f(x)$, then $f^{-1}(y) = x$</p> <p>Remember: Object = the value of x Image = the value of y or $f(x)$ $f(x)$ map onto itself means $f(x) = x$</p>

02 Quadratic Equations

<p>General Form</p> $\boxed{ax^2 + bx + c = 0}$ <p>where a, b, and c are constants and $a \neq 0$.</p> <p>*Note that the highest power of an unknown of a quadratic equation is 2.</p>	<p>Quadratic Formula</p> $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ <p>When the equation can not be factorized.</p>
<p>Forming Quadratic Equation From its Roots:</p> <p>If α and β are the roots of a quadratic equation</p> $\boxed{\alpha + \beta = -\frac{b}{a}} \quad \boxed{\alpha\beta = \frac{c}{a}}$ <p>The Quadratic Equation</p> $\boxed{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$ <p style="text-align: center;">or</p> $\boxed{x^2 - (SoR)x + (PoR) = 0}$ <p>SoR = Sum of Roots PoR = Product of Roots</p>	<p>Nature of Roots</p> <p>$b^2 - 4ac > 0 \Leftrightarrow$ two real and different roots $b^2 - 4ac = 0 \Leftrightarrow$ two real and equal roots $b^2 - 4ac < 0 \Leftrightarrow$ no real roots $b^2 - 4ac \geq 0 \Leftrightarrow$ the roots are real</p>

03 Quadratic Functions

<p>General Form</p> $f(x) = ax^2 + bx + c$ <p>where a, b, and c are constants and $a \neq 0$.</p> <p>*Note that the highest power of an unknown of a quadratic function is 2.</p> <p>$a > 0 \Rightarrow$ minimum $\Rightarrow \cup$ (smiling face)</p> <p>$a < 0 \Rightarrow$ maximum $\Rightarrow \cap$ (sad face)</p>	<p>Completing the square:</p> $f(x) = a(x + p)^2 + q$ <p>(i) the value of x, $x = -p$</p> <p>(ii) min./max. value = q</p> <p>(iii) min./max. point = $(-p, q)$</p> <p>(iv) equation of axis of symmetry, $x = -p$</p> <p><u>Alternative method:</u></p> $f(x) = ax^2 + bx + c$ <p>(i) the value of x, $x = -\frac{b}{2a}$</p> <p>(ii) min./max. value = $f(-\frac{b}{2a})$</p> <p>(iii) equation of axis of symmetry, $x = -\frac{b}{2a}$</p>
<p>Quadratic Inequalities</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>$a > 0$ and $f(x) > 0$</p>  <p>$x < a$ or $x > b$</p> </div> <div style="text-align: center;"> <p>$a > 0$ and $f(x) < 0$</p>  <p>$a < x < b$</p> </div> </div>	<p>Nature of Roots</p> <p>$b^2 - 4ac > 0 \Leftrightarrow$ intersects two different points at x-axis</p> <p>$b^2 - 4ac = 0 \Leftrightarrow$ touch one point at x-axis</p> <p>$b^2 - 4ac < 0 \Leftrightarrow$ does not meet x-axis</p>

04 Simultaneous Equations

To find the intersection point \Rightarrow solves simultaneous equation.

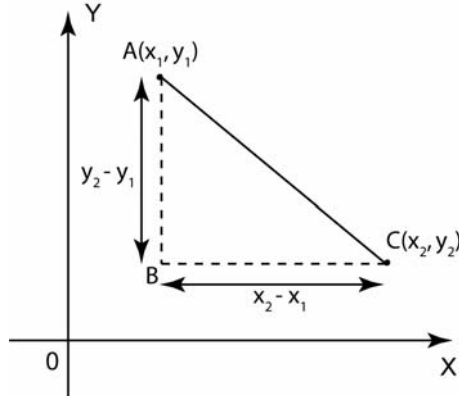
Remember: substitute linear equation into non-linear equation.

05 Indices and Logarithm

<p>Fundamental of Indices</p> <p>Zero Index, $a^0 = 1$</p> <p>Negative Index, $a^{-1} = \frac{1}{a}$</p> <p>$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$</p> <p>Fractional Index $a^{\frac{1}{n}} = \sqrt[n]{a}$</p> <p>$a^{\frac{m}{n}} = \sqrt[n]{a^m}$</p>	<p>Laws of Indices</p> <p>$a^m \times a^n = a^{m+n}$</p> <p>$a^m \div a^n = a^{m-n}$</p> <p>$(a^m)^n = a^{m \times n}$</p> <p>$(ab)^n = a^n b^n$</p> <p>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$</p>
<p>Fundamental of Logarithm</p> <p>$\log_a y = x \Leftrightarrow a^x = y$</p> <p>$\log_a a = 1$</p> <p>$\log_a a^x = x$</p> <p>$\log_a 1 = 0$</p>	<p>Law of Logarithm</p> <p>$\log_a mn = \log_a m + \log_a n$</p> <p>$\log_a \frac{m}{n} = \log_a m - \log_a n$</p> <p>$\log_a m^n = n \log_a m$</p> <p>Changing the Base</p> <p>$\log_a b = \frac{\log_c b}{\log_c a}$</p> <p>$\log_a b = \frac{1}{\log_b a}$</p>

06 Coordinate Geometry

Distance and Gradient



Distance Between Point A and C =

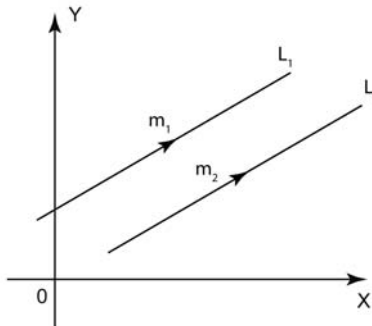
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Gradient of line AC, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Or

Gradient of a line, $m = -\left(\frac{y - \text{int intercept}}{x - \text{int intercept}}\right)$

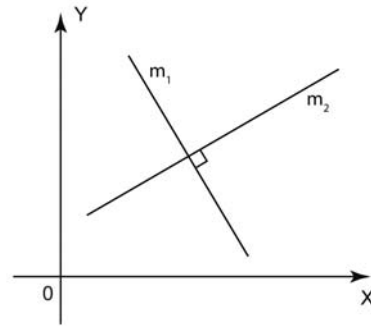
Parallel Lines



When 2 lines are parallel,

$$m_1 = m_2$$

Perpendicular Lines

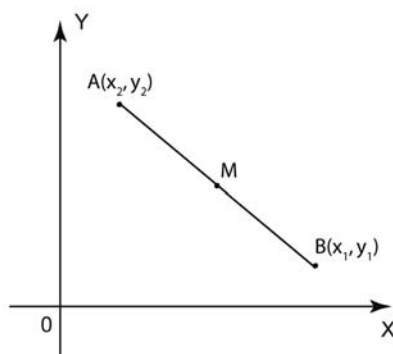


When 2 lines are perpendicular to each other,

$$m_1 \times m_2 = -1$$

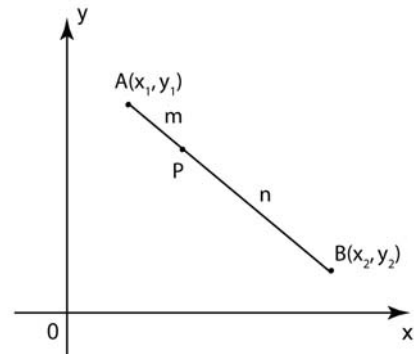
m_1 = gradient of line 1
 m_2 = gradient of line 2

Midpoint



Midpoint, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

A point dividing a segment of a line



A point dividing a segment of a line

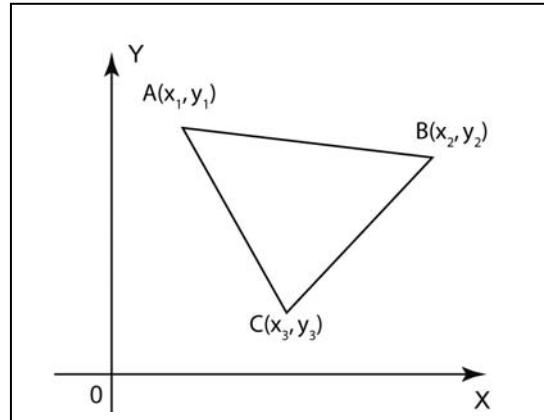
$$P = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

Area of triangle:

Area of Triangle

$$= \frac{1}{2} \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{array} \right|$$

$$A = \frac{1}{2} \left| (x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3) \right|$$



Form of Equation of Straight Line

General form	Gradient form	Intercept form
$ax + by + c = 0$	$y = mx + c$ $m = \text{gradient}$ $c = \text{y-intercept}$	$\frac{x}{a} + \frac{y}{b} = 1$ $a = \text{x-intercept}$ $b = \text{y-intercept}$
		$m = -\frac{b}{a}$

Equation of Straight Line

Gradient (m) and 1 point (x_1, y_1) given $y - y_1 = m(x - x_1)$	2 points, (x_1, y_1) and (x_2, y_2) given $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$	x-intercept and y-intercept given $\frac{x}{a} + \frac{y}{b} = 1$
Equation of perpendicular bisector \Rightarrow gets midpoint and gradient of perpendicular line.		

Information in a rhombus:

	<ul style="list-style-type: none"> (i) same length $\Rightarrow AB = BC = CD = AD$ (ii) parallel lines $\Rightarrow m_{AB} = m_{CD}$ or $m_{AD} = m_{BC}$ (iii) diagonals (perpendicular) $\Rightarrow m_{AC} \times m_{BD} = -1$ (iv) share same midpoint $\Rightarrow \text{midpoint } AC = \text{midpoint } BD$ (v) any point \Rightarrow solve the simultaneous equations
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Remember:

y-intercept $\Rightarrow x = 0$

cut y-axis $\Rightarrow x = 0$

x-intercept $\Rightarrow y = 0$

cut x-axis $\Rightarrow y = 0$

**point lies on the line \Rightarrow satisfy the equation \Rightarrow substitute the value of x and of y of the point into the equation.

Equation of Locus

(use the formula of distance)

The equation of the locus of a moving point $P(x, y)$ which is always at a constant distance (r) from a fixed point $A(x_1, y_1)$ is

$$PA = r$$

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

The equation of the locus of a moving point $P(x, y)$ which is always at a constant distance from two fixed points $A(x_1, y_1)$ and $B(x_2, y_2)$ with a ratio $m : n$ is

$$\frac{PA}{PB} = \frac{m}{n}$$

$$\frac{(x - x_1)^2 + (y - y_1)^2}{(x - x_2)^2 + (y - y_2)^2} = \frac{m^2}{n^2}$$

The equation of the locus of a moving point $P(x, y)$ which is always equidistant from two fixed points A and B is the **perpendicular bisector** of the straight line AB .

$$PA = PB$$

$$(x - x_1)^2 + (y - y_1)^2 = (x - x_2)^2 + (y - y_2)^2$$

More Formulae and Equation List:

[SPM Form 4 Physics - Formulae List](#)

[SPM Form 5 Physics - Formulae List](#)

[SPM Form 4 Chemistry - List of Chemical Reactions](#)

[SPM Form 5 Chemistry - List of Chemical Reactions](#)

All at **One-School.net**

07 Statistics

Measure of Central Tendency

	Ungrouped Data	Grouped Data	
		Without Class Interval	With Class Interval
Mean	$\bar{x} = \frac{\sum x}{N}$ <p> \bar{x} = mean $\sum x$ = sum of x x = value of the data N = total number of the data </p>	$\bar{x} = \frac{\sum fx}{\sum f}$ <p> \bar{x} = mean $\sum x$ = sum of x f = frequency x = value of the data </p>	$\bar{x} = \frac{\sum fx}{\sum f}$ <p> \bar{x} = mean f = frequency x = class mark = $\frac{(\text{lower limit} + \text{upper limit})}{2}$ </p>
Median	$m = \frac{T_{\frac{N+1}{2}}}{2}$ <p>When N is an odd number.</p> $m = \frac{\frac{T_N}{2} + \frac{T_{\frac{N}{2}+1}}{2}}{2}$ <p>When N is an even number.</p>	$m = \frac{T_{\frac{N+1}{2}}}{2}$ <p>When N is an odd number.</p> $m = \frac{\frac{T_N}{2} + \frac{T_{\frac{N}{2}+1}}{2}}{2}$ <p>When N is an even number.</p>	$m = L + \left(\frac{\frac{1}{2}N - F}{f_m} \right) C$ <p> m = median L = Lower boundary of median class N = Number of data F = Total frequency before median class f_m = Total frequency in median class c = Size class = (Upper boundary – lower boundary) </p>

Measure of Dispersion

	Ungrouped Data	Grouped Data	
		Without Class Interval	With Class Interval
variance	$\sigma^2 = \frac{\sum x^2}{N} - \bar{x}^2$	$\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$	$\sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2$
Standard Deviation	$\sigma = \sqrt{\text{variance}}$ $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$ $\sigma = \sqrt{\frac{\sum x^2}{N} - \bar{x}^2}$	$\sigma = \sqrt{\text{variance}}$ $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$ $\sigma = \sqrt{\frac{\sum x^2}{N} - \bar{x}^2}$	$\sigma = \sqrt{\text{variance}}$ $\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$ $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$

The **variance** is a measure of the **mean** for the square of the **deviations** from the **mean**.

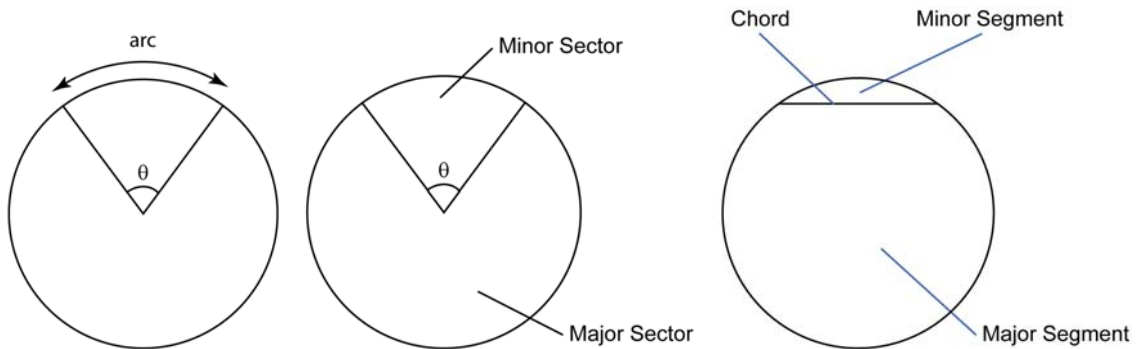
The **standard deviation** refers to the **square root** for the **variance**.

Effects of data changes on Measures of Central Tendency and Measures of dispersion

		Data are changed uniformly with			
		+ k	- k	× k	÷ k
Measures of Central Tendency	Mean, median, mode	+ k	- k	× k	÷ k
	Range , Interquartile Range	No changes		× k	÷ k
Measures of dispersion	Standard Deviation	No changes		× k	÷ k
	Variance	No changes		× k ²	÷ k ²

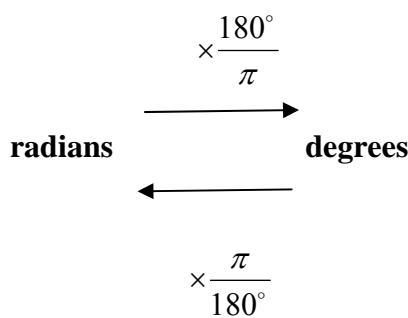
08 Circular Measures

Terminology



Convert degree to radian:

Convert radian to degree:

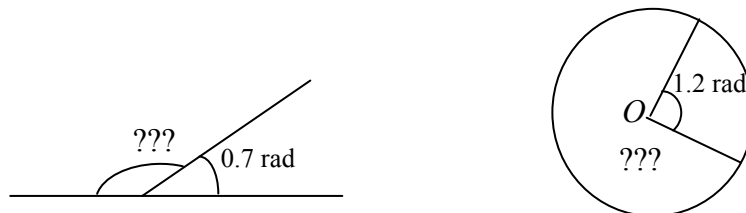


$$x^\circ = (x \times \frac{\pi}{180}) \text{radians}$$

$$x \text{ radians} = (x \times \frac{180}{\pi}) \text{degrees}$$

Remember:

- 180° = π rad
- 360° = 2π rad



Length and Area

Arc Length:	Length of chord:	Area of Sector:	Area of Triangle:	Area of Segment:
$s = r\theta$	$l = 2r \sin \frac{\theta}{2}$	$A = \frac{1}{2}r^2\theta$	$A = \frac{1}{2}r^2 \sin \theta$	$A = \frac{1}{2}r^2(\theta - \sin \theta)$

09 Differentiation

Gradient of a tangent of a line (curve or straight)

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \left(\frac{\delta y}{\delta x} \right)$$

Differentiation of a Function I

$$y = x^n$$

$$\frac{dy}{dx} = nx^{n-1}$$

Example

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

**Differentiation of Algebraic Function
Differentiation of a Constant**

$$y = a \quad a \text{ is a constant}$$

$$\frac{dy}{dx} = 0$$

Example

$$y = 2$$

$$\frac{dy}{dx} = 0$$

Differentiation of a Function II

$$y = ax$$

$$\frac{dy}{dx} = ax^{1-1} = ax^0 = a$$

Example

$$y = 3x$$

$$\frac{dy}{dx} = 3$$

Differentiation of a Function III

$$y = ax^n$$

$$\frac{dy}{dx} = anx^{n-1}$$

Example

$$y = 2x^3$$

$$\frac{dy}{dx} = 2(3)x^2 = 6x^2$$

Differentiation of a Fractional Function

$$y = \frac{1}{x^n}$$

Rewrite

$$y = x^{-n}$$

$$\frac{dy}{dx} = -nx^{-n-1} = \frac{-n}{x^{n+1}}$$

Example

$$y = \frac{1}{x}$$

$$y = x^{-1}$$

$$\frac{dy}{dx} = -1x^{-2} = \frac{-1}{x^2}$$

Chain Rule

$$y = u^n \quad u \text{ and } v \text{ are functions in } x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Example

$$y = (2x^2 + 3)^5$$

$$u = 2x^2 + 3, \quad \text{therefore } \frac{du}{dx} = 4x$$

$$y = u^5, \quad \text{therefore } \frac{dy}{du} = 5u^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 5u^4 \times 4x$$

$$= 5(2x^2 + 3)^4 \times 4x = 20x(2x^2 + 3)^4$$

Or differentiate directly

$$y = (ax + b)^n$$

$$\frac{dy}{dx} = n.a.(ax + b)^{n-1}$$

$$y = (2x^2 + 3)^5$$

$$\frac{dy}{dx} = 5(2x^2 + 3)^4 \times 4x = 20x(2x^2 + 3)^4$$

Law of Differentiation**Sum and Difference Rule**

$$y = u \pm v \quad u \text{ and } v \text{ are functions in } x$$

$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

Example

$$y = 2x^3 + 5x^2$$

$$\frac{dy}{dx} = 2(3)x^2 + 5(2)x = 6x^2 + 10x$$

Product Rule

$$y = uv \quad u \text{ and } v \text{ are functions in } x$$

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Example

$$y = (2x+3)(3x^3 - 2x^2 - x)$$

$$u = 2x+3 \quad v = 3x^3 - 2x^2 - x$$

$$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = 9x^2 - 4x - 1$$

$$\begin{aligned} \frac{dy}{dx} &= v \frac{du}{dx} + u \frac{dv}{dx} \\ &= (3x^3 - 2x^2 - x)(2) + (2x+3)(9x^2 - 4x - 1) \end{aligned}$$

Or differentiate directly

$$y = (2x+3)(3x^3 - 2x^2 - x)$$

$$\frac{dy}{dx} = (3x^3 - 2x^2 - x)(2) + (2x+3)(9x^2 - 4x - 1)$$

Quotient Rule

$$y = \frac{u}{v} \quad u \text{ and } v \text{ are functions in } x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Example

$$y = \frac{x^2}{2x+1}$$

$$u = x^2 \quad v = 2x+1$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = 2$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2} \\ &= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2} \end{aligned}$$

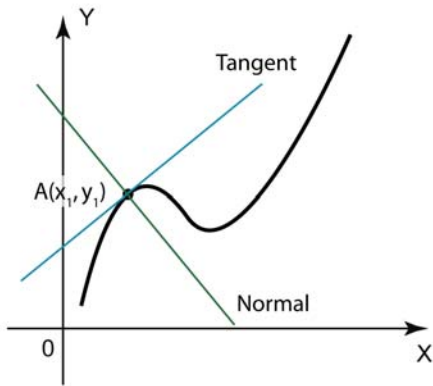
Or differentiate directly

$$y = \frac{x^2}{2x+1}$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$$

Gradients of tangents, Equation of tangent and Normal



Gradient of tangent at A(x₁, y₁):

$$\frac{dy}{dx} = \text{gradient of tangent}$$

Equation of tangent: $y - y_1 = m(x - x_1)$

Gradient of normal at A(x₁, y₁):

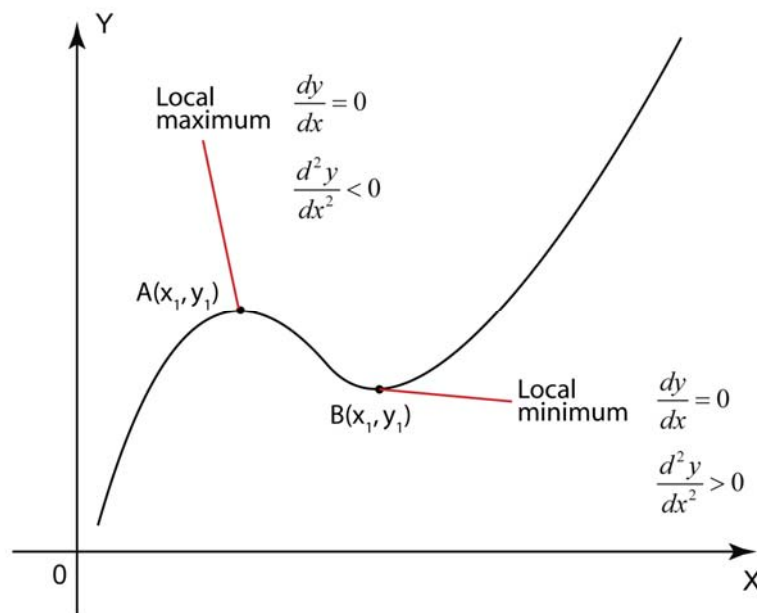
$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}}$$

$$\frac{1}{-\frac{dy}{dx}} = \text{gradient of normal}$$

Equation of normal : $y - y_1 = m(x - x_1)$

If A(x₁, y₁) is a point on a line y = f(x), the gradient of the line (for a straight line) or the gradient of the tangent of the line (for a curve) is the value of $\frac{dy}{dx}$ when x = x₁.

Maximum and Minimum Point



Turning point $\Rightarrow \frac{dy}{dx} = 0$

At maximum point,

$$\frac{dy}{dx} = 0$$

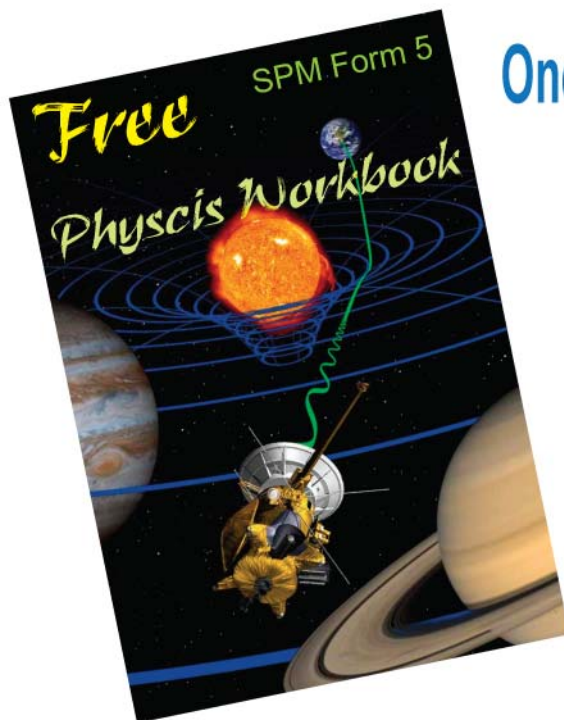
$$\frac{d^2y}{dx^2} < 0$$

At minimum point ,

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} > 0$$

Rates of Change	Small Changes and Approximation
<p>Chain rule $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$</p> <p>If x changes at the rate of $5 \text{ cms}^{-1} \Rightarrow \frac{dx}{dt} = 5$</p> <p>Decreases/leaks/reduces \Rightarrow NEGATIVES values!!!</p>	<p>Small Change:</p> $\frac{\delta y}{\delta x} \approx \frac{dy}{dx} \Rightarrow \delta y \approx \frac{dy}{dx} \times \delta x$ <p>Approximation:</p> $y_{\text{new}} = y_{\text{original}} + \delta y$ $= y_{\text{original}} + \frac{dy}{dx} \times \delta x$ <p>δx = small changes in x δy = small changes in y If x becomes smaller $\Rightarrow \delta x = \text{NEGATIVE}$</p>



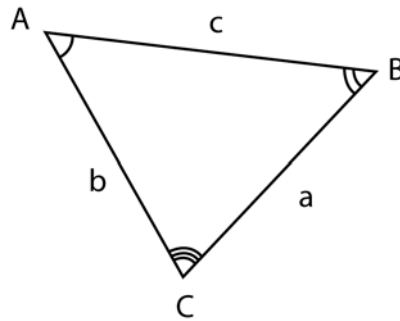
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10 Solution of Triangle

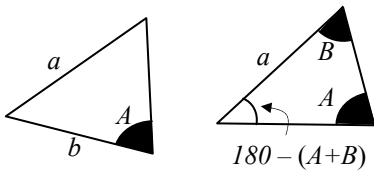


Sine Rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use, when given

- 2 sides and 1 non included angle
- 2 angles and 1 side



Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

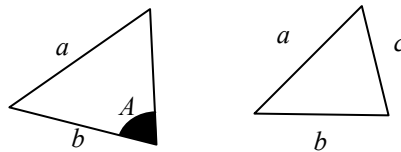
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

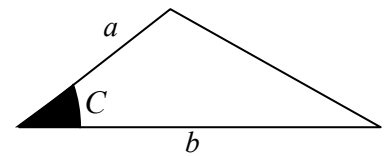
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Use, when given

- 2 sides and 1 included angle
- 3 sides



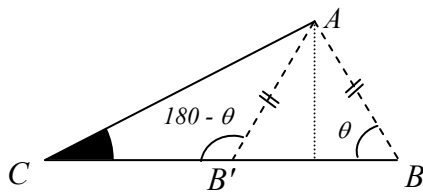
Area of triangle:



$$A = \frac{1}{2} a b \sin C$$

C is the included angle of sides *a* and *b*.

Case of AMBIGUITY



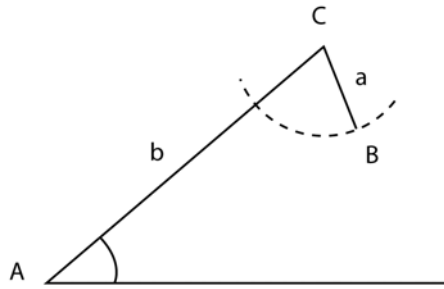
If $\angle C$, the length AC and length AB remain unchanged, the point B can also be at point B' where $\angle ABC = \text{acute}$ and $\angle A B' C = \text{obtuse}$.

If $\angle ABC = \theta$, thus $\angle A B' C = 180 - \theta$.

Remember : $\sin \theta = \sin (180^\circ - \theta)$

Case 1: When $a < b \sin A$

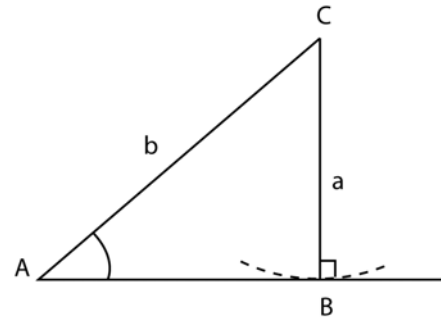
CB is too short to reach the side opposite to C.



Outcome:
No solution

Case 2: When $a = b \sin A$

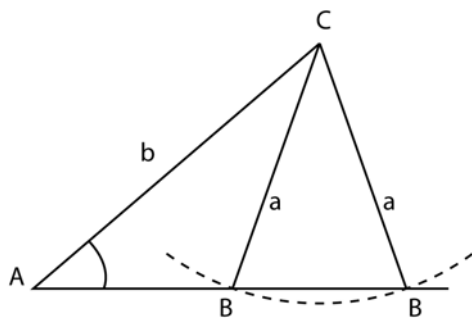
CB just touch the side opposite to C



Outcome:
1 solution

Case 3: When $a > b \sin A$ but $a < b$.

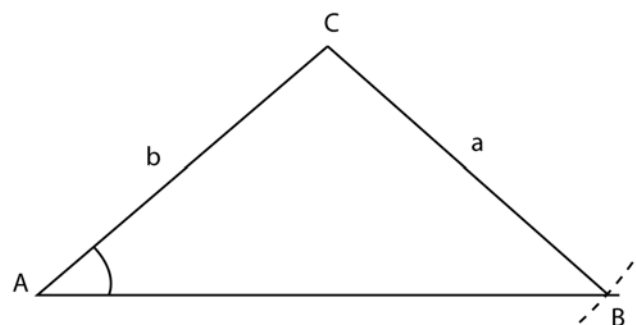
CB cuts the side opposite to C at 2 points



Outcome:
2 solution

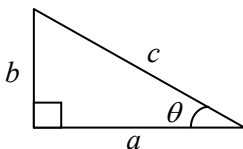
Case 4: When $a > b \sin A$ and $a > b$.

CB cuts the side opposite to C at 1 points



Outcome:
1 solution

Useful information:



In a right angled triangle, you may use the following to solve the problems.

(i) Phythagoras Theorem: $c = \sqrt{a^2 + b^2}$

Trigonometry ratio:

(ii) $\sin \theta = \frac{b}{c}$, $\cos \theta = \frac{a}{c}$, $\tan \theta = \frac{b}{a}$

(iii) $Area = \frac{1}{2} (\text{base})(\text{height})$

11 Index Number

Price Index	Composite index
$I = \frac{P_1}{P_0} \times 100$ <p> <i>I</i> = Price index / Index number <i>P</i>₀ = Price at the base time <i>P</i>₁ = Price at a specific time </p>	$\bar{I} = \frac{\sum W_i I_i}{\sum W_i}$ <p> \bar{I} = Composite Index <i>W</i> = Weightage <i>I</i> = Price index </p>
$I_{A,B} \times I_{B,C} = I_{A,C} \times 100$	



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